

1. Differentiate with respect to x , giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\frac{\sin 4x}{x^3}$

(5)



2.

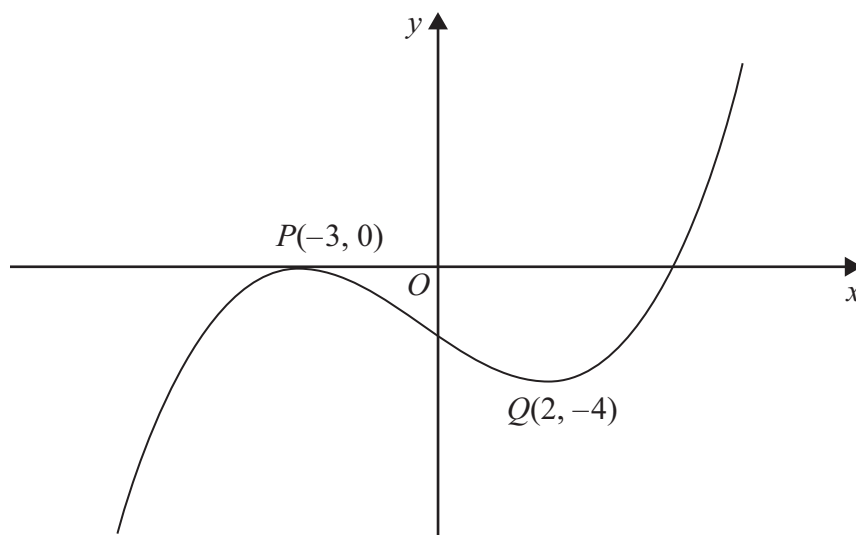


Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3f(x + 2)$

(3)

(b) $y = |f(x)|$

(3)

On each diagram, show the coordinates of any stationary points.



Question 2 continued

Q2

(Total 6 marks)



3. The area, $A \text{ mm}^2$, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20e^{1.5t}, \quad t \geq 0$$

- (a) Write down the area of the culture at midday. **(1)**
- (b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute. **(5)**



4. The point P is the point on the curve $x = 2 \tan\left(y + \frac{\pi}{12}\right)$ with y -coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P .

(7)



6. $f(x) = x^2 - 3x + 2\cos(\frac{1}{2}x), \quad 0 \leq x \leq \pi$

- (a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$ **(2)**

The curve with equation $y = f(x)$ has a minimum point P .

- (b) Show that the x -coordinate of P is the solution of the equation

$$x = \frac{3 + \sin(\frac{1}{2}x)}{2} \quad \text{(4)}$$

- (c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin(\frac{1}{2}x_n)}{2}, \quad x_0 = 2$$

find the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. **(3)**

- (d) By choosing a suitable interval, show that the x -coordinate of P is 1.9078 correct to 4 decimal places. **(3)**



7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$ (4)

(b) Find $f^{-1}(x)$ (3)

(c) Find the domain of f^{-1} (1)

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e . (4)



8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \tag{4}$$

- (b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta} \tag{3}$$

- (c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

Give your answers as multiples of π . (6)



